The Influence of Statistical Fluctuations on Erraticity Behavior of Multiparticle System [1]

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Abstract

It is demonstrated that in low multiplicity sample, the increase of the fluctuation of event-factorial-moments with the diminishing of phase space scale, called "erraticity", are dominated by the statistical fluctuations. The erraticity behavior observed at NA27 experiment can be readily reproduced by pure statistical fluctuations. Applying erraticity analysis to a high multiplicity sample is recommended and the method is improved at very high multiplicity case as well.

Since the finding of unexpectedly large local fluctuations in a high multiplicity event recorded by the JACEE collaboration [1], the investigation of non-linear phenomena in high energy collisions has attracted much attention [2]. The anomalous scaling of factorial moments, defined as

$$F_q = \frac{1}{M} \sum_{m=1}^{M} \frac{\langle n_m(n_m - 1) \cdots (n_m - q + 1) \rangle}{\langle n_m \rangle^q},$$ (1)

at diminishing phase space scale or increasing division number M of phase space [3]:

$$F_q \propto M^{-\phi_q},$$ (2)

called intermittency (or fractal) has been proposed for this purpose in multiparticle system. The average $\langle \cdots \rangle$ in Eqn.(1) is over the whole event sample and n_m is the number of particle falling in the mth bin. That kind of anomalous scaling has been observed successfully in various experiments [4][5].

A recent new development further along that direction is the event-by-event analysis [6][7]. An important step in this kind of analysis was made by Cao and Hwa [8], who first pointed out the difference between a dynamic system in which time sequence can be traced and the multiparticle system where only (phase) space patterns can be obtained. They proposed to measure the pattern by the event factorial moments

$$F_q^{(e)} = \frac{\frac{1}{M} \sum\limits_{m=1}^{M} n_m(n_m - 1) \cdots (n_m - q + 1)}{\left(\frac{1}{M} \sum\limits_{m=1}^{M} n_m \right)^q}$$ (3)

as opposed to sample-factorial-moments defined in Eqn.(1) averaged over all events. Its fluctuations from event to event can be quantified by its normalized moments as:

$$C_{p,q} = \langle \Phi_q^p \rangle, \quad \Phi_q = F_q^{(e)} \Big/ \langle F_q^{(e)} \rangle,$$ (4)

and by $dC_{p,q}/dp$ at $p = 1$:

$$\Sigma_q = \langle \Phi_q \ln \Phi_q \rangle$$ (5)

If there is a power law behavior of the fluctuation as division number goes to infinity, or as resolution $\delta = \Delta/M$ goes to very small, i.e.,

$$C_{p,q}(M) \propto M^{\psi_q(p)},$$ (6)

the phenomenon is referred to as erraticity [9]. The derivative of exponent $\psi_q(p)$ at $p = 1$

$$\mu_q = \frac{d}{dp} \psi_q(p) \Big|_{p=1} = \frac{\partial \Sigma_q}{\partial \ln M}.$$ (7)

describes the width of the fluctuation and so is called as entropy index. In the following, we will call $C_{p,q}$ or Σ_q in Eqn.(4) and (5) as erraticity-moments.

It is well known that the obstacle of event-by-event analysis is the influence of statistical fluctuations caused by insufficient number of particles. The big advantage of sample factorial moments in Eqn.(1) is that it can eliminate this kind of statistical fluctuations. It has been proved [3] that if the statistical fluctuations of particles falling in a bin is Possonian like, then the sample-factorial-moments equal to the corresponding dynamic probability moments:

$$F_q = C_q = M^{q-1} \sum_{1=1}^{M} \langle p_i^q \rangle. \tag{8}$$

Again, the average is over the whole sample. The p_i is the probability of particle falling in the ith bin in a certain event. However, we can not follow the same procedure to get the similar equation for event factorial moments of Eqn.(3). Since the number of particles in an event is not large enough and so is the number of bin, event factorial moments can not completely eliminate statistical fluctuations and present the dynamic probability moments associated with it. How large of the statistical fluctuations in erraticity analysis is and how it depends on the number of multiplicity have not been seriously estimated yet. We will answer these questions quantitatively in the letter.

To be direct and obvious, we firstly use an unique flat probability distribution in whole studying interval and whole sample. It means that the probabilities in all bins are equal and are the same for different events. For simplicity, we use only the fixed number of multiplicity. In this case the denominator in the definition of factorial moment, eqn.(1) and (3), becomes simply $N(N-1)$ [3]. We first take $N = 9$, which is about the average multiplicity at ISR energies. The distribution of particle in the whole studying phase region in an event can be readily located by Bernouli distribution:

$$B(n_1, \ldots, n_M | p_1, \ldots, p_M) = \frac{N!}{n_1! \cdots n_M!} p_1^{n_1} \cdots p_M^{n_M}, \qquad \sum_{m=1}^{M} n_m = N. \tag{9}$$

By this way, we simulate a sample with 1000 events. The results of the second order sample-factorial-moments F_2, erraticity-moments $C_{p,2}$ and Σ_2 on the division number M of the phase-space region are shown in Fig.1(a). The second order sample-factorial-moment is a constant with the increasing of division number. This is what we expect. Since no dynamic is input, it becomes a constant after eliminating the statistical fluctuations. While the increase of erraticity-moments $C_{p,2}$ and Σ_2 with division number is measurably large. These contributions come from pure statistical fluctuations of event factorial moments due to the insufficient number of particle in an event since there is no dynamic fluctuation from event to event in the case. This results can fully recover what has observed in NA27 data [10], cf. the open circles in the second figure of Fig.1(a). This means that in low multiplicity sample, the statistical fluctuations of event factorial moments dominate the erraticity behavior of

multiparticle system. Event factorial moments is not a good representation of event dynamic at low multiplicity events.

However, erraticity analysis proposed a very important way to study the event-by-event fluctuations though we are still not clear whether there are such fluctuations and if there are what mechanism causes them. We have demonstrated in our former paper [12] that if and only if different events have different dynamic fluctuation strengths, the erraticity moments will keep increasing with the increasing of division number and so has nonzero entropy index.

As is well known, the statistical fluctuations will become negligible if the multiplicity of an event is large enough. At how high a multiplicity the event factorial moments can measure the dynamic fluctuations of a finite particle system is a very meaningful question. In the left of the paper, we will focus our discussion on answering the question.

Now we switch the fixed multiplicity N to 20 and 300 in the above mentioned simulation. The corresponding second order sample-factorial-moments F_2, erraticity-moments $C_{p,2}$ and Σ_2 versus the division number M of the phase-space region are shown in Fig.1(b) and (c) respectively. The second order sample-factorial-moment keep to be a constant as we know. The erraticity moments become flater and flater with the increase of multiplicity. It means that pure statistic fluctuations of event factorial moments are greatly depressed by the increase of multiplicity.

From Fig.(1), we can see that, when multiplicity is larger than 300, event factorial moments can be approximately used to describe the event spatial pattern associated with it and its moments — erraticity moments — can represent the erraticity behavior of the system safely.

In order to confirm this upper limit of multiplicity, we do following parallel analysis for a system with dynamic fluctuation from event to event. As we know [12], random cascading model, or α-model is the simplest model which can be used to generate a sample with nonzero entropy index. We will use it for our quantitative discussion below. In the random cascading α-model, the M division of a phase space region Δ is made in steps. At the first step, it is divided into two equal parts; at the second step, each part in the first step is further divided into two equal parts, and so on. The steps are repeated until $M = \Delta Y / \delta y = 2^\nu$. How particles are distributed from step-to-step between the two parts of a given phase space cell is defined by the independent random variable $\omega_{\nu j_\nu}$, where j_ν is the position of the sub-cell $(1 \leq j_\nu \leq 2^\nu)$ and ν is the number of steps. It is given by [11]:

$$\omega_{\nu,2j-1} = \frac{1}{2}(1 + \alpha r) \quad ; \quad \omega_{\nu,2j} = \frac{1}{2}(1 - \alpha r), \qquad j = 1, \ldots, 2^{\nu-1} \qquad (10)$$

where, r is a random number distributed uniformly in the interval $[-1, 1]$. α is a positive number less than unity, which determines the region of the random variable ω and describes the strength of dynamical fluctuations in the model. If it change from event to event, there will be different dynamic fluctuation strength in different events. Here, let it has a Gaussian distribution. The mean and variance of the Gaussian are both chosen as 0.22. After ν steps,

the probability in the mth window $(m = 1, \ldots, M)$ is $p_m = \omega_{1j_1}\omega_{2j_2}\ldots\omega_{\nu j_\nu}$.

By the model, we generate 1000 events. The intermittency analysis, or the logarithm of second order sample probability moment $\ln C_2$, and erraticity moments $\ln C_{p,2}$ and Σ_2 as function of $\ln M$ are shown in Fig.2(a). Now the sample probability moment $\ln C_2$ has a power law behavior as dynamic fluctuations have been input. The erraticity moments also show a power law behavior at large division number region. It represents the dynamic fluctuation from event to event. The corresponding entropy index obtained from a linear fit to the last 3 points of Σ_2 is $\mu_2 = 0.0161$.

Finite number of particle can also be added to the above pure dynamic fluctuation model by Bernouli distribution of Eqn.(9). Again, we put $N = 9$ first. The corresponding factorial moment and erraticity moments are given in Fig.2(c). The value of erraticity moments now are much larger than those obtained from the original pure dynamic fluctuations in Fig.2(a). The entropy index, $\mu_2 = 0.422$, also turns out to be more than one magnitude bigger. This results confirm us again that the erraticity behavior is dominated by statistical fluctuations in low multiplicity sample if we use event factorial moments to characterize it. Though there is dynamic fluctuation from event to event, it will be merged to large statistical fluctuations in the case.

Secondly, we let $N = 300$. The corresponding factorial and erraticity moments are shown in Fig.2(b). The erraticity moments now approach to its original dynamic fluctuation values in Fig.2(a) and entropy index is $\mu_2 = 0.0168$ close to its real value 0.0161. So we get the same conclusion as pure statistic fluctuation case. After multiplicity is larger than 300, erraticity behavior of the system can be estimated by the fluctuation of event factorial moments.

To show quantitatively the influences of statistic fluctuations on erraticity behvior at different multiplicity cases, we simulate various number $N = 5, \ldots, 1000$ of particles in an event for both flat probability distribution and above-described α model cases. The corresponding entropy indices are given in Fig.3 as full circles (flat distribution) and full triangles (α model) respectively. We can see that both of them decrease with multiplicity. For flat probability distribution, entropy index of statistical fluctuation is depressed more than three orders of magnitude when multiplicity N increases from a few to 300. After multiplicity $N > 300$, the entropy index is unmeasurably small. Meanwhile, the entropy index of α-model sample approaches to its real dynamic value $\mu_2 = 0.0161$, cf. the solid line in Fig.2, after $N > 300$. The multiplicity of current and nearly coming heavy-ion collision is about this number or higher. The erraticity analysis given by event factorial moments is applicable for heavy-ion collisions, when the average multiplicity is higher than 300, where we are free from the influence of the statistical fluctuations.

In fact, if multiplicity is higher than a thousand, which has been recorded in NA49 experiments and will be the case for the future heavy-ion collision experiments, the factorial moments analysis of a single event is unnecessary anymore as $n_m(n_m - 1)\cdots$ does not make much difference from $n_m \cdot n_m \cdots$ in most of the phase-space bins which provides main con-

tribution to the anomalous scaling of moments. In these cases the probability distribution in an event can be approximately presented by:

$$p_m \approx \frac{n_m}{N}, \quad 1 = \sum_{m=1}^{M} p_m.$$ (11)

The erraticity-moments of event-probability-moments can be consequently defined by:

$$C_{p,q} = \langle \Phi_q^p \rangle, \quad \Phi_q = \sum_{m=1}^{M} p_m^q \Big/ \langle \sum_{m=1}^{M} p_m^q \rangle$$ (12)

By this definition, we repeat the analysis for both the flat probability distribution and the dynamic-fluctuation distribution cases. It is a little bit smaller than the corresponding event-factorial-moments analysis at flat probability distribution case, cf. the open and full circles in Fig .3, and so it depresses the influence of statistical fluctuations more. The dynamic fluctuations of event to event in the α model case can still be abstracted out as done by the event-factorial-moments description, cf. the open and full triangles in Fig.3.

From the simple discussion above, we can make the following conclusions: If we use event factorial moments to measure spatial pattern, in very low multiplicity sample, such as the sample of ISR energies, statistical fluctuations caused by insufficient number of particle in an event will control the erraticity behavior of the system. Therefore, the physical conclusions from the experimental data on these kind of sample can not be treated seriously. However, if the multiplicity of studying sample is larger than 300, the influence of statistical fluctuations on erraticity behavior is negligible. Therefore, the erraticity behavior, if any, could be well observed in current and future heavy-ion collisions. Further more, if the multiplicity of an event is larger than a thousand, the probability moments defined by Eqn.(11)(12) can present erraticity behavior of the system as well as the event-factorial-moments.

Acknowledgements

References

[1] T. H. Burnett et al., *Phys. Rev. Lett.* **50** (1983) 2062.

[2] E.A. De Wolf, I.M. Dremin and W. Kittel, *Phys. Rep.* **270** (1996) 1.

[3] A. Białas and R. Peschanski, *Nucl. Phys.* B **273** (1986) 703; *ibid* **308** (1988) 857;

[4] N.M. Agababyan et al. (NA22), *Phys. Lett.* **B382** (1996) 305; *ibid* **B431** (1998) 451;

[5] S. Wang, Z. Wang and C. Wu, *Phys. Lett.* B **410** (323) 1997.

[6] A. Białas and B. Ziajia, *Phys. Lett.* B **378** (1996) 319.

[7] Liu Lianshou (EMU01), *Nucl. Phys.* **B** (Proc. Suppl.) 71 (1999) 341 and the references cited therein.

[8] Z. Cao and R. Hwa, *Phys. Rev. Lett.* **75** (1995) 1268; Z. Cao and R. Hwa, *Phys. Rev.* D **54** (1996) 6674; Z. Cao and R. Hwa, *Phys. Rev.* E **56** (1997) 326; Z. Cao and R. Hwa, *Erraticity analysis of soft production by ECOMB*, University of Oregon preprint OITS 666, 1999.

[9] R. C. Hwa, *Acta Physica Polonico* **B27** (1996) 1789.

[10] Wang Shaushun and Wang Zhaomin, *Phys. Rev.* D **57** (1998) 3036.

[11] Wu Yuanfang, Zhang Kunshi and Liu Lianshou, *Chinese Science Bulletin* **36** (1991) 1077.

[12] Fu Jinghua, Wu Yuanfang and Liu Lianshou, *On the intermittency and chaos in high energy collisions*, to be published.

Figure Captions

Fig.1 (a) The dependence of the logarithm of the second order sample-factorial-moments F_2, erraticity-moments $C_{p,2}$ and Σ_2 on that of the phase-space division number M for a flat probability distribution with particle number equal to 9 (a), 20 (b) and 300 (c) respectively. The dashed lines are linear fit. Open points are experimental results of NA27. The solid points are MC results. The solid lines are for guiding the eye.

Fig.2 (a) The dependence of the logarithm of the second order probability-moments C_2, erraticity-moments $C_{p,2}$ and Σ_2 on that of the phase-space division number M for the α model with Gaussian-distributed α. (b) The same as (a) but for the sample-factorial-moments F_2 and the corrssponding erraticity-moments $C_{p,2}$ and Σ_2 with 300 particles. (c) The same as (b) but with 9 particles. The dashed lines are linear fit. The solid lines are for guiding the eye.

Fig.3 The dependence on number of particle of the entropy indices μ_2 for Gaussian α-model calculated from event-factorial-moments (full triangles) and from probability-moments (open triangles). The same for flat distribution (full and open circles). The solid line is the dynamical result without statistical fluctuation. The dashed lines are for guiding the eye.
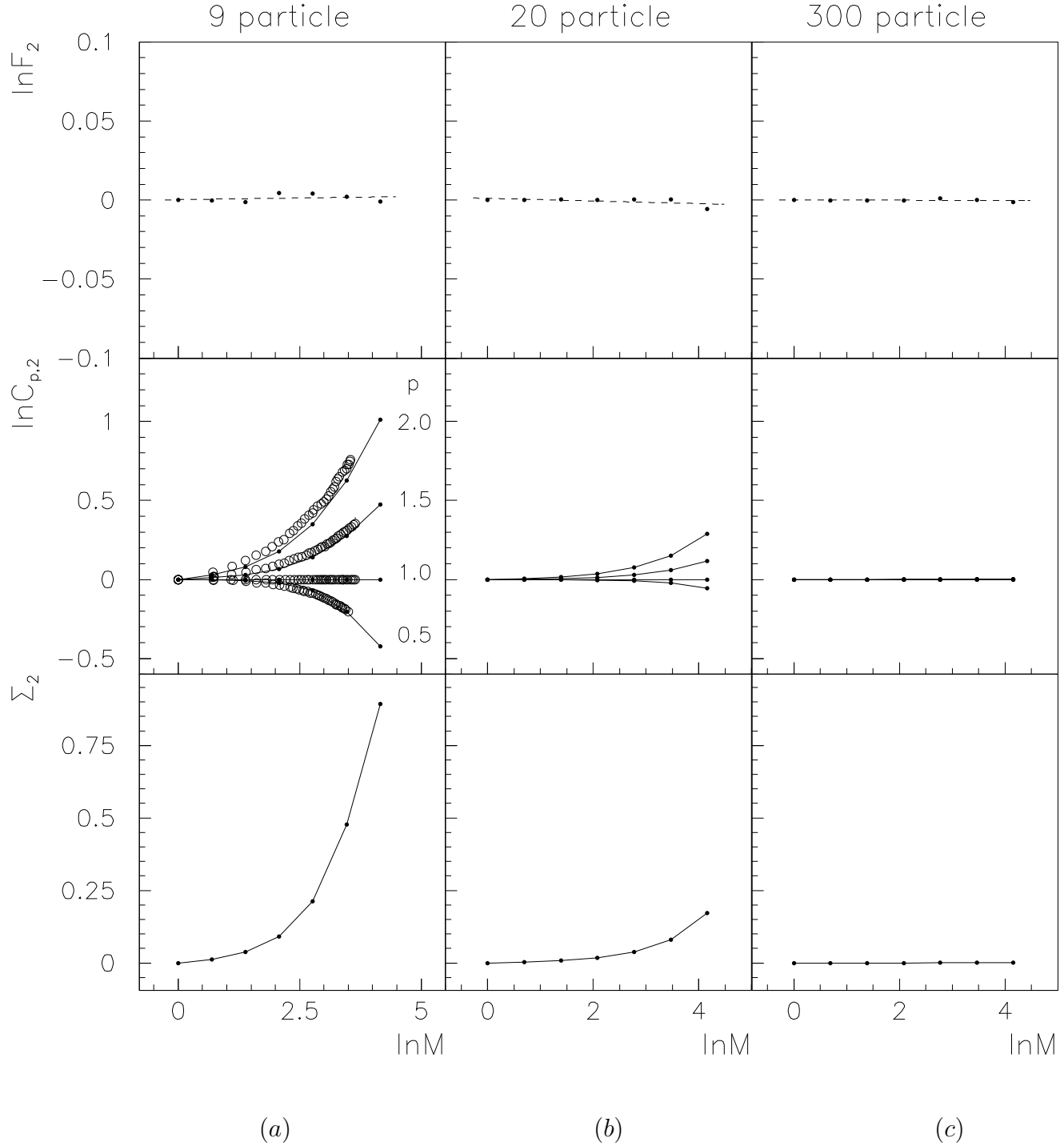
Fig.1 (*a*) The dependence of the logarithm of the second order sample-factorial-
moments F_2, erraticity-moments $C_{p,2}$ and Σ_2 on that of the phase-space
division number M for a flat probability distribution with particle number
equal to 9 (*a*), 20 (*b*) and 300 (*c*) respectively. The dashed lines are linear
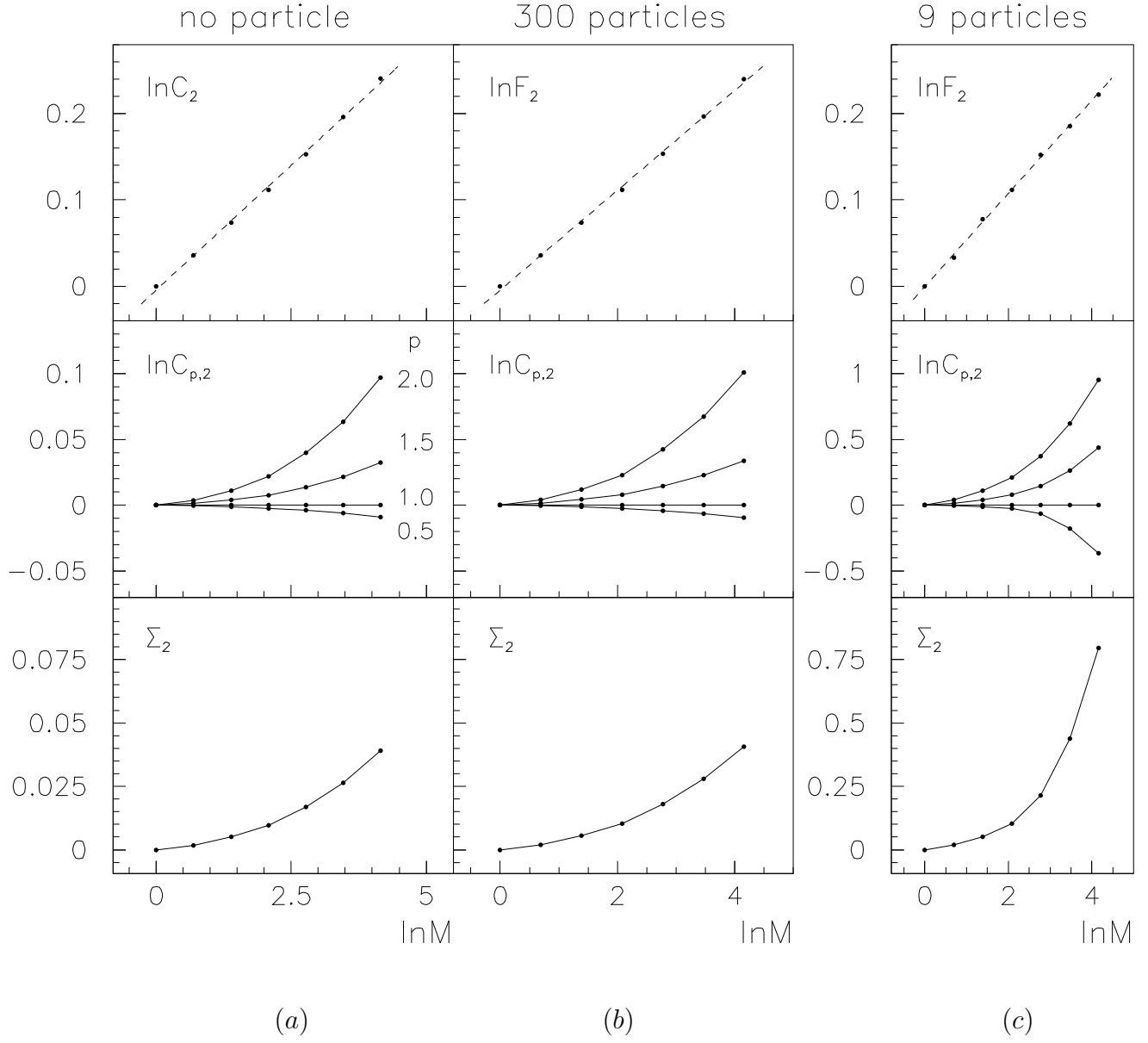fit. The solid lines are for guiding the eye.

Fig.2 (*a*) The dependence of the logarithm of the second order probability-moments C_2, erraticity-moments $C_{p,2}$ and Σ_2 on that of the phase-space division number M for the α model with Gaussian-distributed α. (*b*) The same as (*a*) but for the sample-factorial-moments F_2 and the corrssponding erraticity-moments $C_{p,2}$ and Σ_2 with 300 particles. (c) The same as (*b*) but with 9 particles. The dashed lines are linear fit. The solid lines are for guiding the eye.
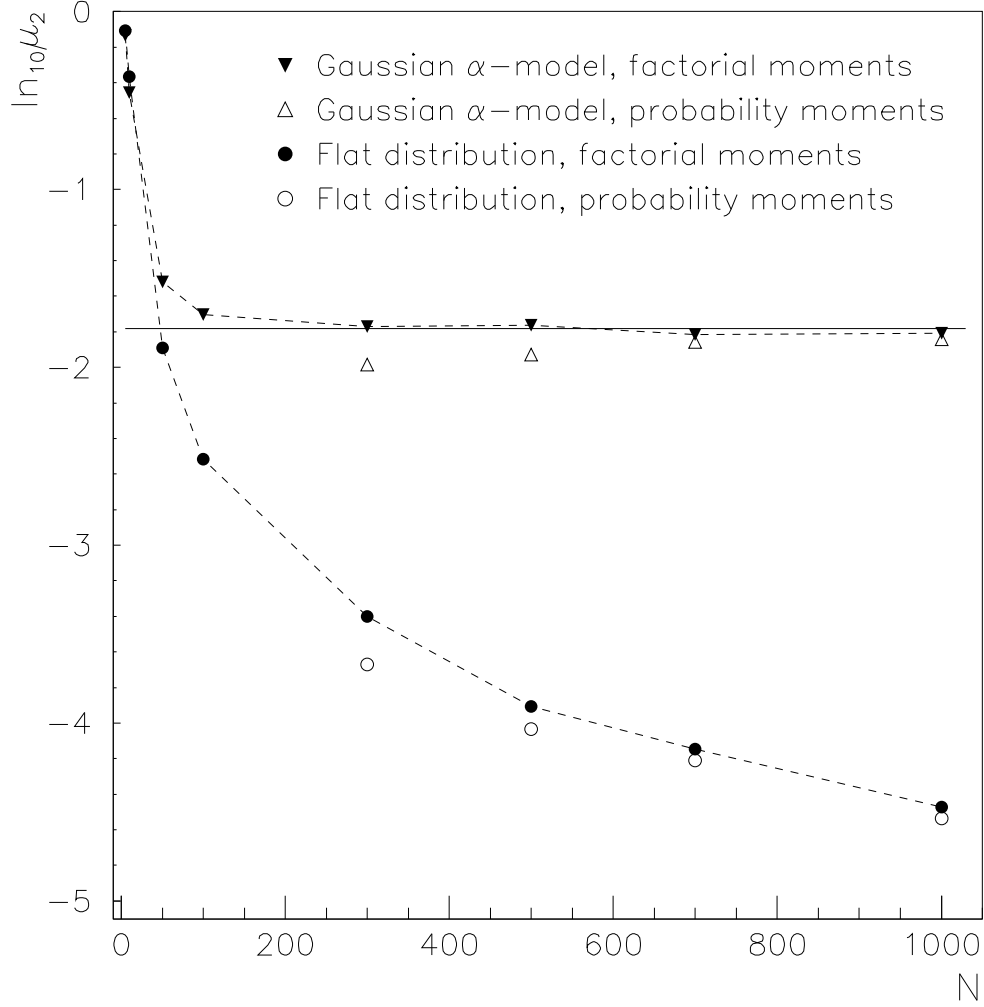
Fig.3 The dependence on number of particle of the entropy indices μ_2 for Gaussian α-model calculated from event-factorial-moments (full triangles) and from probability-moments (open triangles). The same for flat distribution (full and open circles). The solid line is the dynamical result without statistical fluctuation. The dashed lines are for guiding the eye.